# NAG Toolbox for MATLAB

# f03ac

## 1 Purpose

f03ac calculates the determinant of a real symmetric positive-definite band matrix using a Cholesky factorization.

## 2 Syntax

```
[det, rl, ifail] = f03ac(a, n, m, 'm1', m1)
```

## 3 Description

The determinant of $A$ is calculated using the Cholesky factorization $A = LL^{\mathrm{T}}$, where $L$ is a lower triangular band matrix. The determinant of $A$ is the product of the squares of the diagonal elements of $L$.

## 4 References

Wilkinson J H and Reinsch C 1971 *Handbook for Automatic Computation II, Linear Algebra* Springer–Verlag

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **a(lda,m1) – double array**

**lda**, the first dimension of the array, must be at least **n**.

The lower triangle of the $n$ by $n$ positive-definite symmetric band matrix $A$, with the diagonal of the matrix stored in the $(m+1)$th column of the array, and the $m$ subdiagonals within the band stored in the first $m$ columns of the array. Each row of the matrix is stored in the corresponding row of the array. For example, if $n = 5$ and $m = 2$, the storage scheme is

$$\begin{pmatrix} * & * & a_{11} \\ * & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} \\ a_{42} & a_{43} & a_{44} \\ a_{53} & a_{54} & a_{55} \end{pmatrix}.$$

The elements in the top left corner of the array are not used. The following code may be used to assign elements within the band of the lower triangle of the matrix to the correct elements of the array:

```
for i=1:n
  for j=max(1,i-m):i
    a(i,j-i+m+1) = matrix(i,j);
  end
end
```

2: **n – int32 scalar**

$n$, the order of the matrix $A$.

3: **m – int32 scalar**

$m$, the number of subdiagonals within the band of $A$.

### 5.2 Optional Input Parameters

1: **m1 – int32 scalar**

*Default*: The dimension of the arrays **a**, **rl**. (An error is raised if these dimensions are not equal.) the value $m + 1$.

### 5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldrl

### 5.4 Output Parameters

1: **det – double scalar**

The determinant of $A$.

2: **rl(ldrl,m1) – double array**

The lower triangular matrix $L$, stored in the same way as $A$, except that in place of the diagonal elements, their reciprocals are stored.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**ifail** $= 1$

The matrix $A$ is not positive-definite, possibly due to rounding errors.

**ifail** $= 2$

Overflow. The value of the determinant is too large to be held in the computer.

**ifail** $= 3$

Underflow. The value of the determinant is too small to be held in the computer.

## 7 Accuracy

The accuracy of the determinant depends on the conditioning of the original matrix. For a detailed error analysis see page 54 of Wilkinson and Reinsch 1971.

## 8 Further Comments

The time taken by f03ac is approximately proportional to $n(m + 1)^2$.

This function should only be used when $m \ll n$ since as $m$ approaches $n$, it becomes less efficient to take advantage of the band form.

## 9 Example

```
a = [0, 0, 5;
     0, -4, 6;
     1, -4, 6;
     1, -4, 6;
```

```
     1, -4, 6;
     1, -4, 6;
     1, -4, 5];
n = int32(7);
m = int32(2);
[det, rl, ifail] = f03ac(a, n, m)
```

```
det =
   64.0000
rl =
         0         0    0.4472
         0   -1.7889    0.5976
    0.4472   -1.9124    0.6831
    0.5976   -1.9518    0.7385
    0.6831   -1.9695    0.7774
    0.7385   -1.9789    0.8062
    0.7774   -1.9846    1.4790
ifail =
           0
```